Internalizing Environmental Quality in a Simple Endogenous Growth Model

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Abstract

There is an expansive literature on the relationship between income and environmental quality. Some of the literature introduces air pollution into the consumption function while other use it as an input in the production function. A number of papers handle pollution in both the consumption and production functions. This paper falls in the latter category and looks at the impact of environmental policy on consumption and production and the steady state equilbrium.

1 Introduction to Literature on Growth Models.

A number of growth models have sprung up to explain the relationships between per capital GDP growth and the quality of the environment. One of the celebrated findings was by Grossman and Krueger (1995) who showed an inverted bell-shaped relationship between environmental quality and incomes, implying deteriorating environmental quality at the beginning and improved rates at higher income levels. In most of the cases, economies with higher per capita income and better environmental quality usually have more stringent environmental policy. Therefore, the role of environmental policy cannot be overlooked in the shape of this curve.

The Solow-Swan neoclassical growth model utilizes production functions with the constant return to scale and diminishing return to each factor to predict that in the absence of continuing (exogenous) technological change, the per capita growth rate will finally go to zero. Endogenous growth models - Romer (1986, 1990), Lucas (1988), and Rebelo (1991) - set out to explain the relationships between sustainable per capita growth of economies and technological progress. Some key features of these models include endogenization of technological change, increasing returns to physical and human capitals caused by the external effects such as learning-by-doing, and monopolistic competition of new designs. The growth per capita may not cease in such models because the returns to capital accumulation are increasing, rather than diminishing. Government intervention is introduced into the models, and the long-run growth rate depends on the governmental actions.

Lucas (1988) and Romer (1986) analyzed socially optimal and equilibrium growth paths by including the external effects of the average human capital or the spillovers of the total knowledge into the production functions. The new knowledge created by one firm or individual not only improves the productivity but also has positive external effects on the total knowledge in the society, and hence is an externality to other firms. Romer (1990) and Grossman and Helpman (1991) stress the central role of controlled technological change of profit-maximizing agents in modern economic growth. Technology or knowledge is treated as a non-rival and partially excludable good. The investment of private firms in $R\&D$ creates new designs from which the firm could obtain monopolistic rents. On the other hand, it also contributes to the increase of total knowledge stock in the society that would be used for later $R\&D$. Unbounded and endogenous accumulation of new designs or patents evolves exponentially.

Barro (1990) and Barro and Sala-i-Martin (1992) incorporate the public sector and various public services (expenditure) into the endogenous models by analyzing the growth rates and savings rate impacts of government size and tax policy. Most of the theoretic studies applying the neoclassical framework, mainly by d'Arge and Kogiku (1973 and Stiglitz (1974) explore how environmental capacity constrains economic growth, with assumptions that long-run growth rate and the saving rate are exogenously determined. The Overlapping Generations (OLG) models (John and Pecchenino 1994, Marini and Scaramozzino 1995, and Olson and Knapp 1997) are appropriate to investigate the short run growth and how the behavior interactions between consecutive two generations shape the income-environment curve.

Trying to eliminate the deficiency of neoclassical models, recent works have re-examined the growth-environment issues using different approaches, mainly the AK model - Lucas (1988), Romer (1990), and the Shumpeterian models (Aghion and Howitt, 1998). Though various models are used in the literature, most focused on two aspects of growth-environment interactions. First is the sustainability and conditions of sustainability of economic growth that avoids further deterioration of environmental quality. The other aspect is the optimal environmental policy and the implications of policy reform on growth if environmental policy is necessary at all. Smulders and Gradus (1996) developed conditions under which sustained economic growth is compatible with the preservation of environmental quality, and examined the impacts of increased environmental preference on growth rates. By splitting physical capital into productive and pollution-abating capital, Cassou and Hamilton (2000) compare the efficiency of command-and-control instrument and the tax (price) control on the reproducible abatement capital. Built on the works of Barro (1990) and Barro and Sala-i-Martin (1992), Bovenberg and de Mooij (1997) explored the positive impacts of environmental tax reform on economic growth, pollution and welfare in a second-best world. Stokey (1998) developed a model that generates an inverted U-shape curve between per capita income and the environmental quality. It concludes that whether the environmental concerns will limit growth or not depends on the effects of pollution abatement on the long-run rates of return. Few previous papers explicitly analyzed the optimal environmental policy in the presence of other distortionary taxes over time except Cassou and Hamilton (2000).

1.1 Objectives of the Paper

This paper attempts to extend on the work by Cassou and Hamilton(2000) and Smulders and Gradus(1993) on the links between the environment and economic growth, particularly the optimal environmental policy and the policy effects on growth rates and welfare. Using a simplified version of the endogenous growth model with linear production, the paper examines how environmental policy promotes economic growth in the presence of externality. Environmental taxation and to a lesser extent,technical standard, are examined and compared in terms of their long-run effects on growth. This paper diverges from previous analyses by looking at environmental quality as flow variable that is influenced by regular capital and abatement capital. The paper considers a case where environmental quality enters the consumption and production functions, something not applied in most of the literature. For tractable solutions, a simple AK endogenous growth model is used in the analysis in addition to the general specifications of production technology and preferences. The AK model shows constant returns to broad capital. When environmental quality is incorporated, the model shows increasing returns to scale.

The paper considers only steady-state analysis and some discussion of transitional dynamics. The paper concludes that environmental quality curtails or promotes economic growth depending on its evolution over time. Appropriate choices of environmental policy instruments enable the competitive equilibrium growth to be Pareto optimal, hence improving the social welfare. In steady states, the technical standards are constant or independent of the level of economic variables, but the effluent tax is increasing at a rate proportional to the steady-state growth rate of the economy. In addition, the price level in capital market determines both the optimal technical standard and optimal tax rate.

Though the interactions between economy and the environment appear to be complex, it would be interesting to examine them by addressing such questions as the relations between the environmental quality and long-run growth, and how the environmental policy affects the growth rates. Research that endeavors to shed light on these issues will not only be of educational value but also help policy makers grasp the intricacies of national development that is concomitant with improved environmental quality.

Section 2 presents the basic model of endogenous growth with the environmental sector, and then the AK model is used to compare the decentralized competitive equilibrium and social optimal paths in the presence of environmental quality. Section 5 examines the optimal environmental tax on growth and then the conclusion follows with the paper and puts forward the needs of further research that may be of significance.

2 The Model

The economy is assumed to be perfectly competitive in both the goods and capital market, and is composed of one representative consumer and one representative firm. There is only one consumption good and one type of capital. Capital is owned by the consumer, and rented to the firm for production. There is no population growth, and the population size is normalized to one, so the aggregate consumption and capital is equivalent to the per capita consumption and capital level. Production is the only source of pollution. Being indivisible, the general environmental quality contributes to both consumption and production. With these assumptions, the environment-growth model is specified as a maximization problem of the form[1]

$$Max \int_0^\infty \exp^{-\rho t} U(c, E) dt \tag{1}$$

subject to

$$\dot{K} = Y - c = F(K_y, E) - c \tag{2}$$

The utility function is assumed to be from the family of isoelastic[2] utility functions of the general form

$$U(c, E) = \frac{(c^{\upsilon} E^{\omega})^{1-\sigma} - 1}{1 - \sigma}, \sigma \in (0, 1), \upsilon \in (0, 1), \omega \in (0, 1) \tag{3}$$

[1]These variables and those that follow are functions of time, t. This is conveniently left out to reduce clatter.

[2]See Phillipe and Aghion

where $\epsilon = -\frac{U_{cc}(c,E)\,c}{U_c} = 1 - \upsilon(1-\sigma)$ is the elasticity of inter-temporal substitution of consumption (Appendix A). In each period,$U(c, E)$ is the instantaneous utility at time t, ρ is the time discount rate, which influences the consumer's decision over time. Assume the utility function is strictly concave in consumption,c, and the environmental quality ,E ,then $U_c(c, E) = \frac{\upsilon(1-\sigma)(c^{\upsilon(1-\sigma)-1}E^{\omega(1-\sigma)})}{1-\sigma} > 0$,$U_E(c, E) = \frac{\omega(1-\sigma)(c^{\upsilon(1-\sigma)}E^{\omega(1-\sigma)-1})}{1-\sigma} > 0$, and $U_{cc}(c, E) = d^2U(c, E)/dc^2 < 0$. Consumption and environmental quality are assumed to be weakly separable,$U_{cE}(c, E) > 0$.

2.1 Production

On the production side the representative firm is assumed to use Cobb-Douglas technology to produce final output,Y, using productice capital stock K_y, and environmental quality E as inputs.The AK version of production technology is expressed as

$$Y = F(K_y, E) = AK_yE^{\gamma} \tag{4}$$

We make the usual assumptions of positive marginal product of productive capital,$F_{k_y} = \frac{dF}{dK_y} > 0$,positive marginal product of environmental quality, $F_E = \frac{dF}{dE} > 0$,and unambiguous capital-evironmental effect,$F_{K_yE} = \frac{dF_{K_y}}{dE} \neq 0$ [3].

Differentiating equation 4 with respect to time and rearranging we get the following expression.

$$\frac{\dot{Y}}{Y} = \frac{F_{K_y}K_y}{F}\frac{\dot{K}_y}{K_y} + \frac{F_EE}{F}\frac{\dot{E}}{E} \tag{5}$$

Product growth depends on growth in capital and the environmental quality. The model specification is different from most previous works. Firms use capital to produce final output and pollution is a side-product, other than being an input. Environmental quality, which is determined by the pollution level, contributes to production through affecting the quality of capital. The final output in each period is allocated between consumption for current period and capital saving for future

[3]The subscripts stand for the partial derivatives with respect to these variables.

production. The representative individual needs to allocate the capital saving to production and pollution abatement. Under decentralized competitive equilibrium, the capital allocated to the abatement tends to zero, since the environmental quality is exogenous to the agent. Therefore, the intervention of government in promoting pollution abatement plays crucial role to guarantee sustainable economic growth. Government intervention is examined in Section 5.

2.2 Environment

Environmental quality is considered as a flow variable, which is determined by the levels of productive capital stock,K_y ,and abatement capital stock,K_a[4] as shown below.

$$E = E(K_a, K_y) = K_a^{\beta_1} K_y^{-\beta_2} \tag{6}$$

where $0 < \beta_1 < 1$,$0 < \beta_2 < 1$, $E_{K_a K_a} < 0$,$E_{K_y K_y} > 0$, $E_{K_a K_y} < 0$. In this form,β_1 and β_2 are the elasticity of environmental quality in the two types of capital stocks respectively. The environmental quality function satisfies $E_{K_y} < 0$,$E_{K_a} > 0$, and $E_{K_y K_a} \neq 0$. Society must trade-off the use of capital in order to achieve sustainable growth. The potential decline in environmental quality affects both consumption and production .

The growth rate of environmental quality is given by

$$\frac{\dot{E}}{E} = \frac{E_{K_a} K_a}{E} \frac{\dot{K}_a}{K_a} + \frac{E_{K_y} K_y}{E} \frac{\dot{K}_y}{K_y} \tag{7}$$

Elasticities of environmental quality with respect to two types of capital play decisive roles in determining growth rate of environment. In steady states, environmental quality may increase, decrease or stay constant, but steady-state growth is only realized when the elasticities are constant.

Imposing a relationship between the average environmental quality and average emission level in the economy as $E = D^{-}1$, where D is social average emission

[4]Taking environmental quality as a flow variable allows the model to be simpler and more tractable

level, the marginal contribution of pollution on environmental quality is diminishing.

2.3 Sustainable Growth Ignoring Environmental Quality

With decentralized competitive equilibrium environmental quality deteriorates below some critical value. Environmental quality is exogenous to the consumer and firm so there is no incentive for investment in abatement. A benevolent social planner solves the optimal control problem of expressions 1 and 2 in order to ensure sustainable growth.

Therefore dropping the environmental quality variable from the expressions and assuming a simplified isoelastic utility function without E of $U(c) = \frac{c^{1-\epsilon}-1}{1-\epsilon}$ the current-value Hamiltonian is

$$H^c = U(c) + \lambda[F(K_y) - c] \tag{8}$$

and the first order necessary conditions are :

$$U_c - \lambda = 0 \Longrightarrow U_c = \lambda \tag{9}$$

$$\dot{\lambda} - \rho\lambda = -H^c_{K_y} = -\lambda F_{K_y} \tag{10}$$

From condition 9 the co-state variable/shadow price of capital is equal to the marginal utility of consumption. Using condition 9 and the results $U(c) = c^{-\epsilon} \Rightarrow \dot{\lambda} = -\epsilon\dot{c}^{-\epsilon}, U_{cc} = -\epsilon\dot{c}^{-\epsilon-1}$, and condition 10 we get after re-arranging

$$\frac{\dot{c}}{c} = \frac{1}{\epsilon}[F_{K_y} - \rho] \tag{11}$$

where $\epsilon = -\frac{U_{cc}(c,E)\,c}{U_c}$ is defined on page 5

The growth rate of consumption depends on the marginal product of capital,the rate of time preference, and the elasticity of inter-temporal substitution of consumption. The transversality condition for this problem is $\underset{t\to\infty}{lim} \exp^{-\rho t} \lambda_t K_y(t) = 0$, which implies that asymptotically, either the capital accumulation is trivially small, or the marginal valuation of capital,λ, is zero.

2.4 Some Growth Rates and Environmental Quality

Taking the log of E and differentiating with respect to time we get:

$$\frac{\dot{E}}{E} = \beta_1 \frac{\dot{K}_a}{K_a} - \beta_2 \frac{\dot{K}_y}{K_y} \tag{12}$$

This is non-negative if $\frac{\dot{K}_a}{K_a} / \frac{\dot{K}_y}{K_y} \geq \frac{\beta_2}{\beta_1}$. From this relation, the growth rate of environmental quality changes with the growth rates of capital stocks and the elasticities.

The AK version of production technology is expressed as $Y = AK_y E^\gamma = AK_a^{\gamma\beta_1} K_y^{1-\gamma\beta_2}$. It is concave in environmental quality and $0 < \gamma < 0$, where γ is the elasticity of substitution between environmental quality and polluting capital. When γ tends to zero, the production is unaffected by the environment. Substituting the expression for E into Y and differentiating gives the growth rate of final output for AK technology as follows:

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}_y}{K_y} + \gamma \frac{\dot{E}}{E} = \gamma\beta_1 \frac{\dot{K}_a}{K_a} + (1 - \gamma\beta_2) \frac{\dot{K}_y}{K_y} \tag{13}$$

The steady-state growth exists only if the ratio of growth rates of productive capital and abatement capital is constant. The growth rate of output is determined exogenously by the elasticities of production with respect to environmental quality and capitals. The higher the elasticity, γ , the higher the growth rate of output.

The elasticity of marginal rate of substitution between consumption and environmental quality is ω/v . High value of this ratio indicates that consumer has relatively higher preference on environmental quality in utility function, and environmental quality plays more important roles in economic growth. When this ratio is very small, the role of environmental quality in promoting social welfare is negligible. In a competitive market, the firm's objective is to maximize the profit.

$$\pi = pAK_y E^\gamma - r_y K_y - r_a K_a \tag{14}$$

Normalizing the final output price p to unity the first-order condition for the firm is: $r_y = dF/dK_y = AE^\gamma, r_a = dF/dK_a = 0$

From firm's first-order condition, the return to abatement capital is zero. This drives firms to invest zero in pollution abatement. So, under the competitive equilibrium, taking the environmental quality as exogenous, neither the consumer nor the firm has incentives to control pollution.

3 Socially Optimal Path in AK Model

The socially optimal growth paths are achieved by solving the social planner's problem, whose objective is to maximize the representative agent's utility subject budget constraints for each period. In each period of growth, the social planner needs to choose the optimal level of consumption, capital accumulation and capital allocation between production and abatement in order to realize socially optimal growth rates. Given the initial capital level at period zero, if consumption and capital allocation in each period are optimized, the socially optimal growth is then realized.

The difference of social optimum from competitive equilibrium is that environmental quality is endogenous and has to be decided inside the social planner's problem, and the optimal consumption and abatement investment are dictated by the benevolent social planner as well. But in the latter circumstance, consumption and abatement investment decisions are determined in the market through price mechanisms; environmental quality change results from the market equilibrium. The transition equation $\dot{K} = Y - c$, says that output that is not consumed will be saved as capital accumulation and move to the next period for further production and pollution abatement.

The production function of the social planner is expressed as:

$$Y = AK_y E^{\gamma} = AK_a^{\gamma\beta_1} K_y^{1-\gamma\beta_2} \tag{15}$$

Therefore, the social planner's problem is can be expressed using the current-value Hamiltonian below.

$$H^c = U(c, E) + \lambda[AK_a^{\gamma\beta_1}(K - K_a)^{1-\gamma\beta_2} - c] \tag{16}$$

After substituting for E using equation 6 and the fact that $K = K_a + K_y$ the isoelastic utility function is expressed as below.

$$U(c, E) = \frac{[c^{\upsilon} K_a^{\beta_1 \omega} (K - K_a)^{-\beta_2 \ omega}]^{1-\sigma} - 1}{1 - \sigma} \tag{17}$$

The first-order necessary conditions are[5]:

$$H_c^c = \upsilon c^{\upsilon(1-\sigma)-1} [K_a^{\omega\beta_1} (K - K_a)^{-\omega\beta_2}]^{1-\sigma} - \lambda = 0 \tag{18}$$

$$H_{K_a}^c = \omega c^{\upsilon(1-\sigma)} K_a^{\omega\beta_1(1-\sigma)-1} (K - K_a)^{-\omega\beta_2(1-\sigma)-1} [\beta_1 (K - K_a) + \beta_2 K_a] + \lambda A K_a^{\gamma\beta_1 - 1} (K - K_a)^{-\gamma\beta_2} [\gamma\beta \tag{19}$$

$$\dot{K} = A K_y E^{\gamma} - c \tag{20}$$

Condition 18 says that in the optimum the marginal valuation (shadow price) of capital is equal to the marginal utility of consumption in each period. The second condition (19) requires that, in each period, the abatement capital level should be such that the marginal utility of abatement capital equals to the marginal product of abatement capital.

A similar equation as that of Equation 11 can be derived here using a parrallel technique to get the growth rate of consumption in terms of ρ, the elascities of environmetal variable, the growth rate of the environmental quality of equation 12 and output.

$$\frac{\dot{c}}{c} = \frac{1}{\epsilon} [A K_a^{\gamma\beta_1} K_y^{-\gamma\beta_2} \frac{\beta_1}{\beta_1 + \beta_2 K_a / K_y} - \rho + \omega(1 - \sigma)(\beta_1 \frac{\dot{K}_a}{K_a} - \beta_2 \frac{\dot{K}_y}{K_y})] \tag{21}$$

4 Environmental Quality as an Input and Consumption Good

For this case $\gamma \neq 0, \omega \neq 0$ and when making decisions on optimal consumption and optimal abatement investment in each period, the social planner still needs to

[5]FOCs utilize the fact that utility function is isoelastic

trade-off. In one period, the social planner knows capital saving from last period. If more abatement capital is used, environmental quality increases which benefits the consumer more, nevertheless less polluting capital is left for production, thus less output for consumption, which causes utility to decrease. How much the social planner should earmark for production and abatement is addressed through the first-order conditions of optimal control.

From equation 21 the sufficient and necessary conditions for a steady state of consumption growth to exist are the elasticities of environmental quality of the two types of capital stocks are equal, and the abatement-polluting capital ratio maintains constant. In the steady state, the balanced growth indicates that the growth rates of consumption, capital stocks and output are constant, and the saving rate K/Y, consumption rate c/Y, and capital allocation ratio $\mu = K_a/K_y$ are constant, and environmental quality E maintains constant. In addition assuming the elasticities of environmental qualities for the two capitals are equal then steady-state growth rates are;

$$\frac{\dot{c}}{c} = \frac{1}{\epsilon}[\frac{A\mu^{\gamma\beta}}{1+\mu} - \rho] \tag{22}$$

$$\frac{\dot{K}}{K} = \frac{A\mu^{\gamma\beta}}{1+\mu} - \frac{c}{(1+\mu)K_y} \tag{23}$$

The consumption rate becomes now looks like this.

$$\frac{c}{Y} = \frac{\upsilon[-\gamma\beta + (1-\gamma\beta)\mu]}{\omega\beta(1+\mu)} \tag{24}$$

The growth rate of consumption is determined by inter-temporal elasticity of substitution of consumption, marginal product of capital, and discount rate and capital allocation ratio. The marginal rate of substitution between polluting capital and abatement capital in production function is $MRS = (1/\gamma\beta - 1)\mu$, so μ reflects the MRS between the two capitals. Its effect on growth rate is ambiguous depending on the elasticity values γ and β. When $A\mu^{\gamma\beta}/(1+\mu) > \rho$, the steady growth rate is positive, which says if the marginal product of capital must exceeds discount rate, the economy grows postively. Larger γ means a higher elasticity of production with respect to environmental quality, and larger β means a higher elasticity of environmental quality with respect to capital. From equations 22 and 23 we could observe that if $\mu < 1$, the higher the two elasticities, the lower the steady-state growth rate of consumption, thus the slower the growth rate of the economy. This demonstrates that investment in pollution abatement curtails the steady-state economic growth. Setting equation 22 equal to 23 and using equation 24 to solve for c/K , one is able to obtain the optimal steady-state abating-polluting capital ratio μ[6] From the positive growth condition and the optimal value of μ , the restriction on parameters is further imposed.

5 Environmental Policy and Growth

This section discusses how optimal environmental policies of internalizing pollution promote economic growth, and the behavior patterns of the representative agent under regulations. Private agents have no incentive to invest in pollution abatement in a competitive environment. Thus the abatement capital accumulation is zero. In the competitive equilibrium, environmental quality grows at a negative rate, and jumps to zero. Certainly, this is not socially optimal because of the existence of externality arising from the use of dirty capital in the process of growth. Public choice methods could be used, such as the voting system in the overlapping generation model analyzed in Jones and Manuelli (2000). But the basic regime of governmental intervention is still either the taxes (subsidies) or the technical standards to ensure that the decentralized competitive equilibrium is socially optimal. The objective is to give the private agents incentives in order to replicate socially optimal growth paths under competitive equilibrium.

There are several methods of taxation to improve the investment in the pollution control. Examples include the final output tax, capital tax, emission tax and

[6] The equation to solve for the optimal is non-linear

abatement subsidy. In contrast, the government could also design a technical standard to ensure the investment in pollution abatement i.e. maintain a fixed ratio between abatement and dirty capital.

In addition, one interesting case is to explore whether the tax and technical standard generate the same growth rate in the long run and compare their effects on welfare (utility of representative agents). A dynamic competitive equilibrium is defined as a set of allocations of consumption and capital (c, K_y, K_a) given the prices (p, r_y, r_a) , environmental quality dynamics, the production technology $(Y : Y = AK_yE^{\gamma}, K_y \geq 0, E \geq 0)$ and governmental regulations (μ, τ, s) along the growth path, under which consumer maximizes the present value of utility under the allocations, the firm maximizes the current value of profit in each period, and the goods market, abatement capital market and productive capital market clear.

If environmental policy is able to make competitive equilibrium replicate social optimum, the policy is said to be a first-best one, meaning that policy improves social welfare by fully internalizing pollution impacts. In a second-best environment, the competitive equilibrium is not Pareto optimal, but may be a Pareto improvement realized through policy implementation.

5.1 The environmental (Pigouvian) tax

Due to the inadequate time to cover both technical standards and taxation issues, this paper will delve into the latter and suspend the former for some future appropriate opportunity. Suffice to say that the former deals with maintaining some fixed ratio between abatement and dirty capital over the growth period. For this model, environmental quality is determined by pollution from production, and $E = D^{-1}$ where D is the emission level of identical firms. This indicates that the marginal rate of environment deterioration is diminishing with the pollution amount. Following this postulation, the emission level of the firm is expressed as $D = K_a^{-\beta_1} K_y^{\beta_2}$.

Suppose the regulator wants to put an emission tax with a flat rate τ. At a first look, the tax effect on the growth is ambiguous since, by the emission equation, the tax is on both polluting capital and abatement capital. The following analysis will show whether this intuition is correct.

Under the emission tax, the firm maximizes profit $\pi = Y - r_y K_y - r_a K_a - \tau K_y^{\beta_2} K_a^{-\beta_1}$ where Y is as indicated in Section 3 equation 15 on page 10. The first order condition for firm is:

$$r_y = AE^{\gamma} - \tau \beta_2 K_y^{\beta_2 - 1} K_a^{-\beta_1} \tag{25}$$

$$r_a = \tau \beta_1 K_y^{\beta_2} K_a^{-\beta_1 - 1} \tag{26}$$

Under the emission tax, firms rent capital for production up to a point till the polluting capital rental rate equals the marginal production value of capital minus marginal tax of polluting capital. In abatement capital market, firms buy the capital with an amount when the rental rate equals the marginal tax of abatement capital only since firms do not use abatement capital for production.

In the firm's side, the level of capital rented is determined by marginal values of production and marginal tax of polluting capital, while in the consumer's side, optimality requires that the two capitals have the same price. In the competitive equilibrium, equation 25 equals equation 26 which gives the optimal emission tax rate and the optimal capital allocation ratio:

$$r_a = \frac{A\beta_1 K_a^{\gamma\beta_1} K_y^{-\gamma\beta_2}}{\beta_1 + \beta_2 (K_a/K_y)} \tag{27}$$

$$\tau^* = \frac{AK_a^{1+(1+\gamma)\beta_1} K_y^{1-(1+\gamma)\beta_2}}{\beta_1 K_y + \beta_2 K_a} \tag{28}$$

The capital price is exogenous and changes in each period, so the capital allocation ratio is also changing depending on the capital price. The optimal emission tax rate is the function of capital allocation ratio, hence also depends on capital prices by equation 28

Equation 27 shows how the optimal capital allocation is determined by the capital rent level, which is non-linear in the ratio of capitals. This solution requires the use of approximation methods. For each level of capital rent, there may exist more than one pair of optimal capital allocations depending on the parametric specifications, from which the firm would choose one pair that would maximize profit. From 27 we obtain the optimal capital allocations, then the optimal tax rate is derived by plugging 27 into 28

From the consumer and firm's optimal conditions, following competitive equilibrium paths are obtained:

$$\frac{\dot{c}}{c} = \frac{1}{\epsilon}[r_y - \rho + \omega(1-\sigma)\frac{\dot{E}}{E}] \tag{29}$$

$$\frac{\dot{K}}{K} = r_y - \frac{\dot{c}}{K} \tag{30}$$

5.2 Revisiting Environmental Quality as an Input of Production and a Consumption Good

As described under section 4 this implies that $\gamma \neq 0, \omega \neq 0$. If$\beta_1 = \beta_2 = \beta$, steady-state growth paths in competitive equilibrium are possible. Since the capital rental rate is exogenous to competitive equilibrium, the capital allocation ratio is constant corresponding to each capital rent level. It is observed again that the steady-state capital accumulation rate exists when the consumption-saving ratio is constant.

Therefore, a properly designed emission tax enables the competitive equilibrium to be Pareto optimal when the equilibrium price reaches certain levels, and in this sense, emission tax is also a first-best policy.

Heuristically, when the elasticities of environmental quality in the two types of capital are equal, the ratio of two types of capital stock maintains constant in the sense that capital rent is exogenous to this system. Hence steady-state growth rates of consumption and capital exist as in the social optimum. No matter whether the environmental quality affects utility or not, the optimal tax rate in steady state is proportional to the capital stock, which shows that the tax rate also grows at a constant rate same as the economic growth rate. The existence of all the steady state growths of capital depends on the level of consumption-capital ratio. This imposes the stability problem of steady states. If the consumption-ratio is not able to reach a constant level in any period, the economy only has transitional dynamics, and the competitive equilibrium is not Pareto optimal.

For the regulator, the tax policy should be designed such that it grows proportional to or same as the growth rate of capital. Once it observes the capital accumulation level at each period, it could easily calculate a tax rate and does not need to know how the firm would abate the pollution.

In the environmental tax researches, there is an issue of "double dividend", the opinions on which are not uniform among economists. Proponents of double dividend argue that environmental tax has two forms of virtue as an environmental tax not only to settle the negative externality but also generates tax revenue that contributes to the reduction of distortionary taxes. If an exogenous income tax is put on the consumer or the firm, it is necessary to investigate how the income tax and environmental tax interact. For the general situations where the steady state does not concern, it is not easy to see the tax interactions. When the environmental quality is a production input, it appears that the tax interaction is ambiguous.

In case where E contributes only to consumption and by adding income tax, we obtain the optimal environmental tax:

$$\tau^* = \frac{r_a[\beta_1(1-\tau_y)A - \beta_1 r_a]^{1+\beta_1} K_y^{1+\beta_1-\beta_2}}{\beta_1(\beta_2 r_a)^{1+\beta_1}} \tag{31}$$

It is perceived that higher income tax causes a lower emission tax, even makes it negative. In other words, taxation does help to reduce other distortionary taxes when the environmental quality contributes only to consumption.

5.3 Some Key Issues

From above analysis, we could see that the environmental policy plays an important role in the sustainable growth of economy and environment. Just by imposing environmental regulations on the economy, a Pareto optimal situation is reached through which the social welfare is maximized. However, the implementation problems exist in terms of information availability, implementing costs and other aspects.

The Pigouvian tax also has transitional problems as the optimal tax rate is not easily calculable because it requires that levels of capital stocks the firm uses be known. Even in the steady states, the optimal tax rate is not constant, but increases with the same rate as capital accumulation. This causes that the optimal tax rate to be inconsistent over time.

To the extent that the regulator has to know the firm's capital uses level, the technical standard and Pigouvian tax are equivalent not only in theory but also in implementation.

If we look at the optimal tax rates, it is seen that the optimal rate is determined by the price level in capital markets. Here two problems arise: one is the time lag of policy design and implementation, the other is the time consistency of policy. The government may change the tax rate unexpectedly due to the potentially volatile capital market swings. This imposes implementation difficulty of environmental policy.

6 Conclusion and Issues for Further Research

This paper explores the interactions of environmental quality and social welfare in the process of economic growth, using a simple endogenous growth model. It argues that environmental quality curtails the economic growth if there is no abatement activity in the society. Environmental policy is shown to be conducive to economic growth other than curtailing growth given that it is properly designed and implemented. In the long rund tax instruments are able to move the market equilibrium growth paths to socially optimal ones. However, there exist several problems and difficulties in implementing the proposed environmental policies in this paper.

There are two important aspects related to the topics discussed in this paper that are not examined here at all. First, the technological change is not embodied in the analysis. This is a significant omission, since new knowledge contributes much to pollution control in economies. One interesting question is how the environmental policy could contribute to the incentives for technological changes that are environmentally sound.

Second, one point of endogenous growth models is that (positive) externalities in economy are part of engine of economic growth, such as knowledge spillovers. In recognizing that pollution is a negative externality, how this and positive externalities interact in economic growth is significant problem to be examined. The critical issue is how the environmental quality enters the model.

The interactions between environment and economic growth is complex and the understanding of economists on this issue is far from satisfactory, in part because the physical interactions are still not fully recognized in natural sciences.

Appendix A

From equation 3 on page 5 we can differentiate with respect to consumption and environmental quality to get the following results:

$$U_c(c, E) = \frac{\upsilon(1-\sigma)(c^{\upsilon(1-\sigma)-1}E^{\omega(1-\sigma)})}{1-\sigma} > 0 \tag{32}$$

$$U_E(c, E) = \frac{\omega(1-\sigma)(c^{\upsilon(1-\sigma)}E^{\omega(1-\sigma)-1})}{1-\sigma} > 0 \tag{33}$$

$$U_{cc}(c, E) = d^2U(c, E)/dc^2 = \frac{(\upsilon(1-\sigma)-1)\upsilon(1-\sigma)(c^{\upsilon(1-\sigma)-2}E^{\omega(1-\sigma)})}{1-\sigma} \tag{34}$$

The elasticity of inter-temporal substitution of consumption is given by

$$\epsilon = -\frac{U_{cc}(c, E)\ c}{U_c} = 1 - \upsilon(1-\sigma) \tag{35}$$

References

Aghion, P., and Howitt, P., Endogenous Growth Theory. *Massachusetts: MIT Press* 1998.

Barro, R. J., and Sala-i-Martin, X. Economic Growth. *New York: MacGraw-Hill* 1995.

Grossman, G. M., and Helpman, E. Innovation and Growth in the Global Economy. *Cambridge, Massachusetts: MIT Press* 1991.

Romer, P. M. "The Origins of Endogenous Growth." *Journal of Economic Perspectives* **volume** (8)(winter 1994): 3-22.

Barro, R. J. "Government Spending in a Simple Model of Endogenous Growth." *Journal of Political Economy* **volume** (98)(October 1990): S103-26.

Barro, R. J., and Sala-i-Martin, X. "Public Finance in Models of Economic Growth." *Review of Economic Studies* **volume** (59)(October 1992): 645-61.

Bovenberg, A. L., and de Mooij, R. A. "Environmental Tax Reform and Endogenous Growth." *Journal of Public Economics* **volume** (63)(January 1997): 207-37.

Cassou, S. P., and Hamilton, S. F. "Pollution Abatement Regulation, Property Rights and the Political Economy." *manuscript, Department of Agricultural Economics, the University of Arizona* 2000.

d'Arge, R.C., and Kogiku, K.C. "Economic Growth and the Environment." *Review of Economic Studies* **volume** (40) (January 1973): 61-77.

Elbasha, E. H., and Roe, T. L. "On Endogenous Growth: The Implications of Environmental Externalities." *Journal of Environmental Economics and Management* **volume** (31)(September 1996): 240-268.

Gradus, R., and Smulders, S. "The Trade-Off between Environmental Care and Long-Term Growth–Pollution in Three Prototype Growth Models." *Journal of Economics (Zeitschrift fur Nationalokonomie)* **volume** (58) n1 (1993): 25-51.

Grossman, G. M., and Krueger, A. B. "Economic Growth and the Environment." *Quarterly Journal of Economics* **volume** (110) (May 1995): 353-77.

John, A., and Pecchenino, R. "An Overlapping Generations Model of Growth and The Environment." *The Economic Journal* **volume** (104)(November 1994): 1393-1410.

Jones, L. E., and Manuelli, R. E. "A Convex Model of Equilibrium Growth: Theory and Policy Implications." *Journal of Political Economy* **volume** (98) (October 1990): 1008-1038.

Jones, L. E., Manuelli, R. E., and Rossi, P.E. "Optimal Taxation in Models of Endogenous Growth." *Journal of Political Economy* **volume** (101) (June 1993): 485-517.

Jones, L. E., and Manuelli, R. E. "Endogenous Policy Choice: The Case of Pollution and Growth". *Working Paper, Departments of Economics, Northwestern University and University of Wisconsin-Madison*, May 2000. Available HTTP: http://www.ssc.wisc.edu/ manuelli/research/research.html.

Lucas, R. E., Jr. "On the Mechanics of Economic Development." *Journal of Monetary Economics* **volume** (22)(July 1988): 3-42.

Olson, L. J., and Knapp, K. C. "Exhaustible Resource Allocation in an Overlapping Generations Economy." *Journal of Environmental Economics and Management* **volume** (32)(March 1997): 277-92.

Romer, P. M. "Increasing Returns and Long-Run Growth." *Journal of Political Economy* **volume** (94) (October 1986): 1002-1037.

Romer, P. M. "Endogenous Technological Change." *Journal of Political Economy* **volume**(98)(October 1990): S71-102.

Smulders, S., and Gradus, R. "Pollution abatement and long-term growth." *European Journal of Political Economy* **volume** (12)(November 1996): 505-532.

Stiglitz, J. E. "Growth with Exhaustible Natural Resources: The Competitive Economy." *Review of Economic Studies* **volume** (41)(Symposium 1974): 139-152.

Stokey, N. L. "Are There Limits to Growth?" *International Economic Review* **volume** (39) (February 1998): 1-31.

Rebelo, S. "Long-Run Policy Analysis and Long-Run Growth" *Journal of Political Economy* **volume) (99) (June 1991): 500-521.**

Marini, G., and Scaramozzino, P. "Overlapping Generations and Environmental Control".*Journal of Environmental Economics and Management* volume (29) (July 1995): 64-77.